Synthesis from Weighted Specifications with Partial Domains over Finite Words

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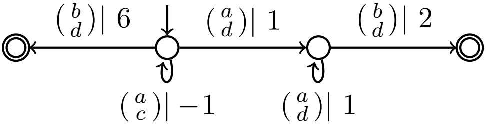
Synthesis

Specification - - - synthesize - - -> **Implementation**

one input is in relation with several outputs

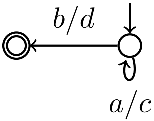
given by a **deterministic synchronous weighted automaton**



$\mathsf{Sum}(\left(\begin{smallmatrix} a & a & b \\ c & d & d \end{smallmatrix}\right)) = -1 + 1 + 2 = 2$
$\mathsf{Sum}(aab \otimes cdd) = 2$

selects unique output for each input

realized by a **sequential synchronous transducer**



Is this **good**?

What are High Quality Implementations?

An implementation is a set of valid executions.

Possible quality constraints

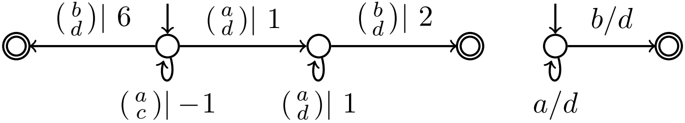
- All executions have a lower bounded quality.
- All executions are quality optimal.
- All executions are almost quality optimal.

Threshold Synthesis

The **threshold synthesis problem** asks, given $c \in \mathbb{Q}$, and $\triangleright \in \{>, \geq\}$, that the implem. f satisfies for all valid inputs u:

$$\mathsf{val}(u \otimes f(u)) \triangleright c$$

Example. Sum-**specification and Implementation**



Implementation ensures value of at least 3 for all pairs.
$\mathsf{Sum}(b \otimes d) = 6, \quad \mathsf{Sum}(a^i b \otimes d^{i+1}) = i \cdot 1 + 2$

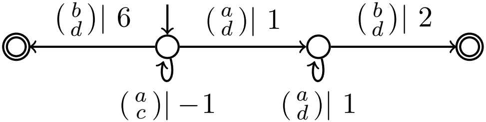
Best-value Synthesis

The **best-value synthesis problem** asks that the implementation f satisfies for all valid inputs u:

$$\mathsf{val}(u \otimes f(u)) = \mathsf{bestVal}(u) := \sup_v \mathsf{val}(u \otimes v),$$

that is, the maximal value achievable for input u.

Example. Sum-**specification**



$\mathsf{bestSum}(b) = 6$
$\mathsf{bestSum}(ab) = 5$
$\mathsf{bestSum}(aab) = 4$
$\mathsf{bestSum}(aaab) = 5$
$\mathsf{bestSum}(aaaab) = 6$

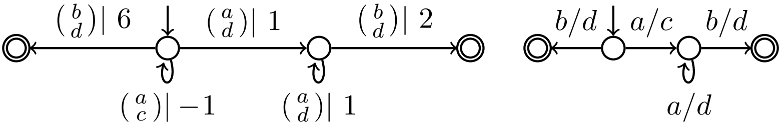
No best-value implementation exists.

Approximate Synthesis

The **approximate synthesis problem** asks, given $c \in \mathbb{Q}$, and $\triangleleft \in \{<, \leq\}$, that the implem. f satisfies for all valid inputs u:

$$\mathsf{bestVal}(u) - \mathsf{val}(u \otimes f(u)) \triangleleft c$$

Example. Sum-**specification and Implementation**



Implementation ensures value of at most 2 less the best value.

$\mathsf{Sum}(b \otimes d) = 6 \qquad \mathsf{bestSum}(b) = 6$

$\mathsf{Sum}(ab \otimes cd) = 5 \qquad \mathsf{bestSum}(ab) = 5$

$\mathsf{Sum}(a^i b \otimes c^i d) = i \qquad \mathsf{bestSum}(a^i b) = i + 2, \quad \text{for } i \geq 2$

Results

Spec / Problem	Sum-automata	Avg-automata	Dsum-automata
strict threshold	NP ∩ coNP	NP ∩ coNP	NP
non-strict threshold	NP ∩ coNP	NP ∩ coNP	NP ∩ coNP
best-value	PTIME [AKL10]	PTIME [AKL10]	NP ∩ coNP
strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	NEXPTIME for discount $1/n$
non-strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	EXPTIME for discount $1/n$

Benjamin Aminof, Orna Kupferman, and Robby Lampert.
Reasoning about online algorithms with weighted automata.
ACM Trans. Algorithms, 6(2):28:1–28:36, 2010.

Emmanuel Filiot, Ismaël Jecker, Nathan Lhote, Guillermo A. Pérez, and Jean-François Raskin.
On delay and regret determinization of max-plus automata.
In *LICS*, pages 1–12. IEEE Computer Society, 2017.