Synthesis from Weighted Specifications with Partial Domains over Finite Words

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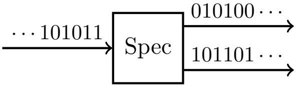
Synthesis

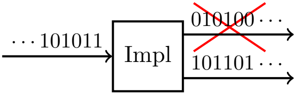
synthesize

Specification - - - - - - - - - → **Implementation**

one input is in relation
with several outputs

selects unique output
for each input

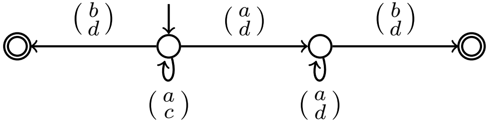




Boolean Specifications

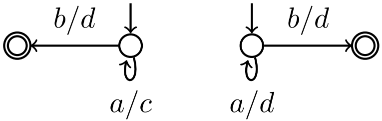
A (Boolean) **specification** is given by a synchronous deterministic automaton.

Example.



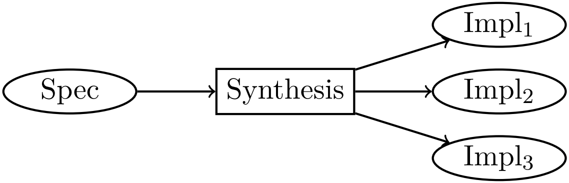
An **implementation** is given by a synchronous sequential transducer.

Example.

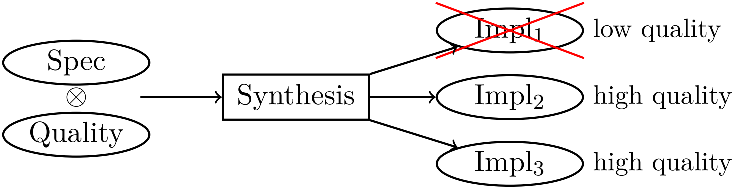


Which one is better?

Quality in Synthesis



A **quality measure** is a function $Q\colon (\Sigma \times \Gamma)^* \to \mathbb{Q}$.



How to Define High Quality?

- All executions have a lower bounded quality:

$$\forall i_1 o_1 i_2 o_2 \cdots \in [\mathrm{Impl}]\colon Q\left(\begin{smallmatrix} i_1 & i_2 & \cdots \\ o_1 & o_2 & \cdots \end{smallmatrix}\right) \geq c$$

- All executions are quality optimal:

$$\forall i_1 o_1 i_2 o_2 \cdots \in [\mathrm{Impl}]\colon Q\left(\begin{smallmatrix} i_1 & i_2 & \cdots \\ o_1 & o_2 & \cdots \end{smallmatrix}\right) = \sup_{o_1' o_2' \cdots} Q\begin{pmatrix} i_1 & i_2 & \cdots \\ o_1' & o_2' & \cdots \end{pmatrix}$$

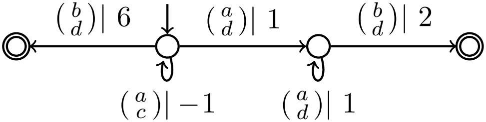
- All executions are almost quality optimal:

$$\forall i_1 o_1 i_2 o_2 \cdots \in [\mathrm{Impl}]\colon \sup_{o_1' o_2' \cdots} Q\begin{pmatrix} i_1 & i_2 & \cdots \\ o_1' & o_2' & \cdots \end{pmatrix} - Q\left(\begin{smallmatrix} i_1 & i_2 & \cdots \\ o_1 & o_2 & \cdots \end{smallmatrix}\right) \leq c$$

Weighted Specifications (:= Boolean Spec $\otimes$ Quality)

A **weighted specification** is a function $\mathsf{val}\colon (\Sigma \times \Gamma)^* \to \mathbb{Q} \cup -\infty$ given by a synchronous deterministic weighted automaton.

Example.



The value val of a pair depends on the used **payoff function**.

Example. $\mathsf{Sum}(\binom{a\ a\ b}{c\ d\ d}) = -1 + 1 + 2 = 2$, $\quad \mathsf{Sum}(aab \otimes cdd) = 2$

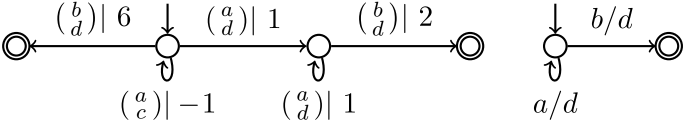
The specification **domain** is $\{u \mid \mathsf{val}(u \otimes v) \in \mathbb{Q}\}$. An input is **valid** if it is from the domain.

Example. Specification domain $= a^*b$

The **threshold synthesis problem** asks, given $c \in \mathbb{Q}$, and $\triangleright \in \{>, \geq\}$, that the implem. f satisfies for all valid inputs u:

$$\mathsf{val}(u \otimes f(u)) \triangleright c$$

Example. Sum-**specification and Implementation**



Implementation ensures value of at least 3 for all pairs.
$\mathsf{Sum}(b \otimes d) = 6, \quad \mathsf{Sum}(a^i b \otimes d^{i+1}) = i \cdot 1 + 2$

Note: Implementation can do anything on invalid inputs.

Threshold Synthesis

Spec / Problem	Sum-automata	Avg-automata	Dsum-automata
strict threshold	$\mathrm{NP} \cap \mathrm{coNP}$	$\mathrm{NP} \cap \mathrm{coNP}$	NP
non-strict threshold	$\mathrm{NP} \cap \mathrm{coNP}$	$\mathrm{NP} \cap \mathrm{coNP}$	$\mathrm{NP} \cap \mathrm{coNP}$

How to solve? See it as a game problem.

We introduce a new type of game.

Critical prefix games

Critical Prefix Games

Tailored to handle

- finite inputs
- partial specification domains

A **critical prefix game** is an **infinite-duration** two-player turn-based **weighted game** with **critical vertices**.

- When a play is in a critical vertex, quantitative constraints on the prefix are checked,
  - if fulfilled, the play continues, otherwise Adam wins.
- Nothing checked for non-critical vertices.

Critical Prefix Games and Threshold Synthesis

Threshold synthesis reduces to critical prefix games with threshold conditions.

These games are decidable for sum, average, and discounted-sum payoffs.

- Sum and average critical prefix games reduce to mean-payoff games.
- Discounted-sum critical prefix games reduce to discounted-sum games.

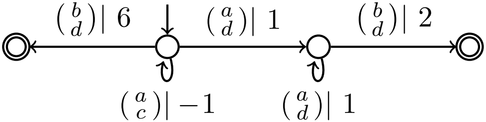
Best-value Synthesis

The **best-value synthesis problem** asks that the implementation f satisfies for all valid inputs u:

$$\mathsf{val}(u \otimes f(u)) = \mathsf{bestVal}(u) := \sup_v \mathsf{val}(u \otimes v),$$

that is, the maximal value achievable for input u.

Example. Sum-**specification**



$$\begin{aligned} \mathsf{bestSum}(b) &= 6 \\ \mathsf{bestSum}(ab) &= 5 \\ \mathsf{bestSum}(aab) &= 4 \\ \mathsf{bestSum}(aaab) &= 5 \\ \mathsf{bestSum}(aaaab) &= 6 \end{aligned}$$

No best-value implementation exists.

Best-value Synthesis

Problem \ Spec	Sum-automata	Avg-automata	Dsum-automata
best-value	PTIME [AKL10]	PTIME [AKL10]	NP ∩ coNP

Proof Techniques

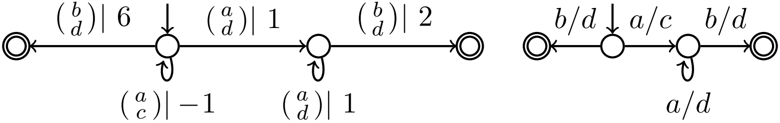
- Sum: reduces to determinization by pruning of Sum-automata
- Avg: reduces to Sum
- Dsum: reduces to a discounted-sum game

Approximate Synthesis

The **approximate synthesis problem** asks, given $c \in \mathbb{Q}$, and $\triangleleft \in \{<, \leq\}$, that the implem. f satisfies for all valid inputs u:

$$\mathsf{bestVal}(u) - \mathsf{val}(u \otimes f(u)) \triangleleft c$$

Example. Sum-**specification and Implementation**



Implementation ensures value of at most 2 less the best value.

$$\begin{array}{ll} \mathsf{Sum}(b \otimes d) = 6 & \mathsf{bestSum}(b) = 6 \\ \mathsf{Sum}(ab \otimes cd) = 5 & \mathsf{bestSum}(ab) = 5 \\ \mathsf{Sum}(a^i b \otimes c^i d) = i & \mathsf{bestSum}(a^i b) = i + 2, \quad \text{for } i \geq 2 \end{array}$$

Approximate Synthesis

Problem \ Spec	Sum-automata	Avg-automata	Dsum-automata
strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	NEXPTIME for discount $1/n$
non-strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	EXPTIME for discount $1/n$

Proof Techniques

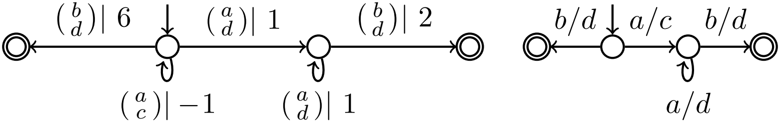
- Sum: reduces to regret determinization of Sum-automata
- Dsum: open in general, for integer discounts reduces to discounted-sum games
- Avg: best-value synthesis reduces to Sum, no longer the case for approximate synthesis

Approximate Synthesis with Average Payoff

$$\mathsf{bestAvg}(u) - \mathsf{Avg}(u \otimes v) \lhd c \quad \Leftrightarrow \quad \frac{\mathsf{bestSum}(u)}{n} - \frac{\mathsf{Sum}(u \otimes v)}{n} \lhd c$$
$$\Leftrightarrow \quad \mathsf{bestSum}(u) - \mathsf{Sum}(u \otimes v) \lhd c \cdot n$$

$$\mathsf{bestAvg}(u) - \mathsf{Avg}(u \otimes v) = 0 \quad \Leftrightarrow \quad \mathsf{bestSum}(u) - \mathsf{Sum}(u \otimes v) = 0$$

Example. Specification and Implementation



$$\mathsf{Sum}(a^i b \otimes c^i d) = i \qquad \mathsf{bestSum}(a^i b) = i + 2, \quad \text{for } i \geq 2$$
$$\mathsf{Avg}(a^i b \otimes c^i d) = \tfrac{i}{2i+2} \qquad \mathsf{bestAvg}(a^i b) = \tfrac{i+2}{2i+2}, \quad \text{for } i \geq 2$$

Reduces to (a special type of) **critical prefix games with imperfect information** which reduce to imperfect information games with fixed initial credit.

Results

Problem \ Spec	Sum-automata	Avg-automata	Dsum-automata
strict threshold	NP ∩ coNP	NP ∩ coNP	NP
non-strict threshold	NP ∩ coNP	NP ∩ coNP	NP ∩ coNP
best-value	PTIME [AKL10]	PTIME [AKL10]	NP ∩ coNP
strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	NEXPTIME for discount $1/n$
non-strict approximate	EXPTIME-c [FJL+17]	decidable EXPTIME-hard	EXPTIME for discount $1/n$

- Benjamin Aminof, Orna Kupferman, and Robby Lampert.
  Reasoning about online algorithms with weighted automata.
  ACM Trans. Algorithms, 6(2):28:1–28:36, 2010.
- Emmanuel Filiot, Ismaël Jecker, Nathan Lhote, Guillermo A. Pérez, and Jean-François Raskin.
  On delay and regret determinization of max-plus automata.
  In *LICS*, pages 1–12. IEEE Computer Society, 2017.