Synthesis from Weighted Specifications over Finite Words

Synthesis Problem

Specification ⟶ (synthesize) ⟶ Implementation

one input is in relation with serveral outputs

selects unique output for each input
same domain as specification

given by
deterministic weighted synchronous automata

realized by
sequential synchronous transducers

Threshold Synthesis Problem

Given a specification S, a threshold $\nu \in \mathbb{Q}$, and $\triangleright \in \{>, \geq\}$, the implementation f has to satisfy

$$S(u \otimes f(u)) \triangleright \nu$$

for all $u \in \mathrm{dom}(S)$.

Best-value Synthsis Problem

Given a specification S, the implementation f has to satisfy

$$S(u \otimes f(u)) = \mathsf{bestVal}_S(u),$$

where $\mathsf{bestVal}_S(u) = \sup\{S(u \otimes v) \mid u \otimes v \in S\}$ for all $u \in \mathrm{dom}(S)$.

Approximate Synthesis Problem

Given a specification S, a threshold $\nu \in \mathbb{Q}$, and $\triangleleft \in \{<, \leq\}$, the implementation f has to satisfy

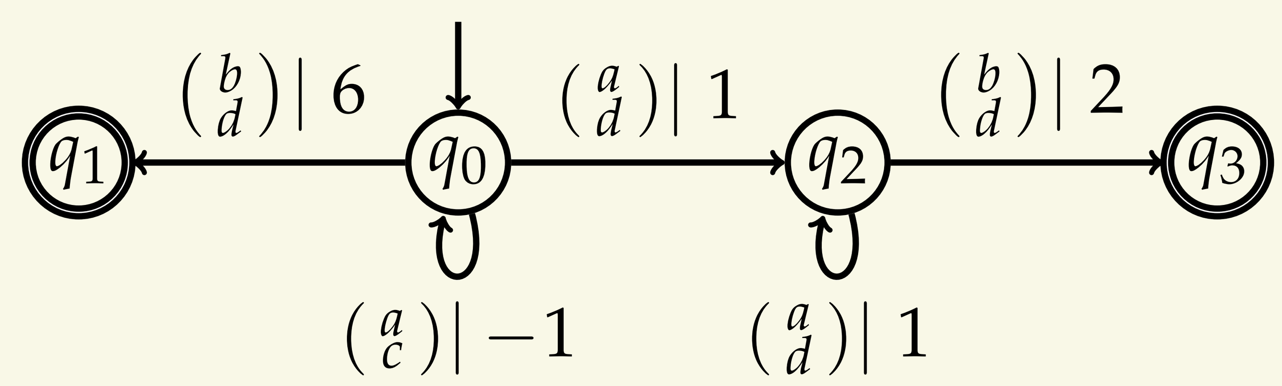
$$\mathsf{bestVal}_S(u) - S(u \otimes f(u)) \triangleleft \nu$$

for all $u \in \mathrm{dom}(S)$.

Proof techniques: Two-player games

- Imperfect information energy games to obtain results on approximate synthesis for Avg-specs.
- Mean-payoff and discounted sum games for other results.

Example: Sum-specification

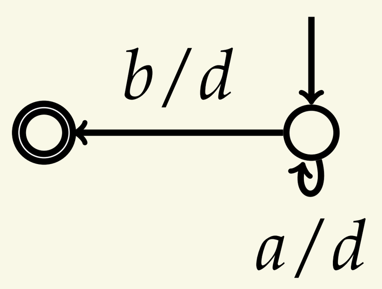


$\mathsf{Sum}\left(\begin{pmatrix} a & a & b \\ c & d & d \end{pmatrix}\right) = -1 + 1 + 2 = 2, \quad S(aab \otimes cdd) = 2$

$\mathsf{Sum}\left(\begin{pmatrix} a & a & b \\ c & c & d \end{pmatrix}\right) = -1 - 1 + 6 = 4, \quad S(aab \otimes ccd) = 4$

Example: Threshold

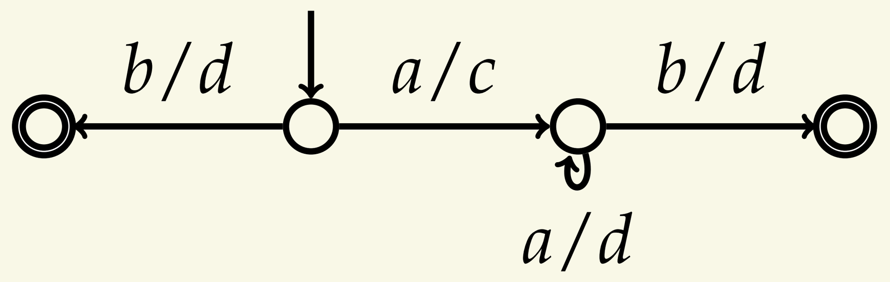
A value-≥ 3-implementation



$S(b \otimes d) = 6, \quad S(a^i b \otimes d^{i+1}) = i \cdot 1 + 2$

Example: Best-value & approximate

No best-value implementation exists, but a ≤ 2-approximate implementation.



$S(b \otimes d) = 6$	$\mathsf{bestVal}(b) = 6$
$S(ab \otimes cd) = 5$	$\mathsf{bestVal}(ab) = 5$
$S(aab \otimes cdd) = 2$	$\mathsf{bestVal}(aab) = 4$
$S(aaab \otimes cddd) = 3$	$\mathsf{bestVal}(aaab) = 5$
$S(aaaab \otimes cdddd) = 4$	$\mathsf{bestVal}(aaaab) = 6$

Results

Problem \ Specification	Sum-automata	Avg-automata	Dsum-automata
strict threshold	NP $\cap$ coNP	NP $\cap$ coNP	NP†
non-strict threshold	NP $\cap$ coNP	NP $\cap$ coNP	NP $\cap$ coNP
best-value	Ptime [AKL10]	Ptime [AKL10]	NP $\cap$ coNP†
strict approximate	EXPtime-c [JFL$^+$17]	decidable†	NEXPtime for discount factor $1/n$
non-strict approximate	EXPtime-c [JFL$^+$17]	decidable†	EXPtime for discount factor $1/n$

† most involved results

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